

Criterion B Task

This section is assessed against criterion B only.

You should spend 50-60 minutes on this section, and you are advised to make full use of your GDC.

The transpose of matrix M is written as M^T and it is constructed in a very simple way – you write the rows as columns instead.

For example, if $M = \begin{pmatrix} 1 & -6 & 3 \\ 0 & 9 & -8 \end{pmatrix}$ then $M^T = \begin{pmatrix} 1 & 0 \\ -6 & 9 \\ 3 & -8 \end{pmatrix}$

Part I: Getting Used to Transposes

(a) If $A = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$ what is A^T ?

(b) If $B = \begin{pmatrix} 1 & 3 & -1 \\ 6 & 6 & 5 \\ -3 & 7 & 1 \end{pmatrix}$ what is B^T ?

(c) What is the relationship between the **order** of M and the order of M^T ? (for example, if M is a 3×5 matrix, what is M^T ?)

Part II: Concentrating on Square Matrices

(d) Write down any non-zero 2×2 matrix of your choosing, call it C , and find its determinant.

(e) Find the determinant of C^T .

(f) Write down a different 2×2 matrix, call it D , and find its determinant.

(g) Find the determinant of D^T .

(h) What do you notice about your answers to (d) to (g)?

(i) Can you come up with a general rule that describes what is happening in (d) to (g).

(j) Now prove your general rule in part (i).

Part III: Multiplying Transposes

(k) Ahmed is a Maths student who is investigating this aspect of matrices. He says the following about 2×2 matrices:

$$\text{If } A \text{ and } B \text{ are both } 2 \times 2 \text{ matrices then } (AB)^T = A^T B^T$$

By using an example of your choosing, show that Ahmed is wrong.

(l) Although Ahmed is wrong, there **actually** is a relationship between $(AB)^T$, A^T and B^T . **Find** that relationship.

(m) **Prove** the relationship you came up with in part (l) above, explaining your methods clearly.

(n) **Predict** what would happen to 3×3 matrices C & D using your relationship and **verify** this with one example.